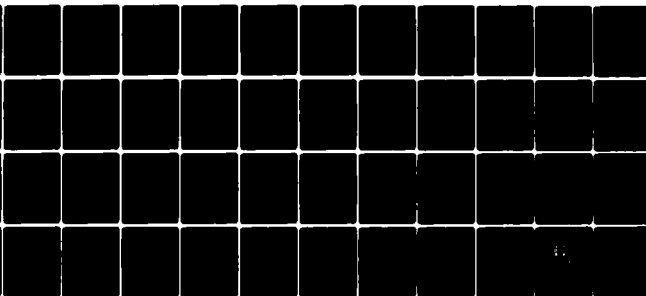


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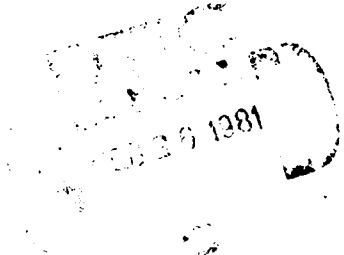
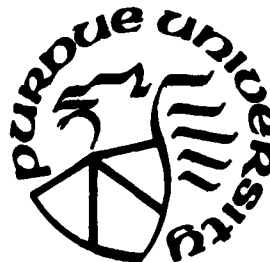
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LEVEL II
**HYBRID APPROACHES AND
INDUSTRIAL APPLICATIONS OF
PATTERN RECOGNITION**

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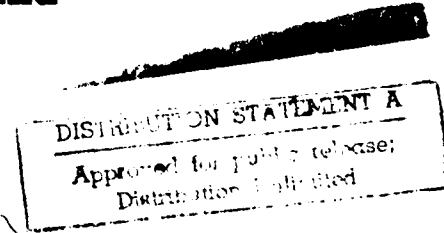
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This work was supported by NATO Research Grant 1639 and
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HYBRID APPROACHES AND INDUSTRIAL APPLICATIONS OF
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ABSTRACT

This report summarizes the major progress made during the support of NATO Research Grant 1639. Chapter I was written by K. S. Fu, Chapter II by J. Kittler and L. F. Pau, and Chapter III by L. F. Pau.

- Chapter I On Hybrid Approaches to Pattern Recognition;
Chapter II Automatic Inspection by Lots in the Presence of Classification Errors;
Chapter III Visual Screening of Integrated Circuits for Metallization Faults by Pattern Analysis Methods.

K. S. Fu is also supported by the ONR Contract N00014-79-C-0574.

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CHAPTER I

ON HYBRID APPROACHES TO PATTERN RECOGNITION

1.1 Introduction

There are many methods proposed for designing a pattern recognition system. These methods can primarily be grouped into two major approaches; namely, decision-theoretic or discriminant approach [1-9], and syntactic or structural approach [10-12]. From a more general viewpoint, these approaches can be discussed within the same framework in terms of pattern representation and decision-making (based on a given pattern representation). A block diagram of a pattern recognition system, based on this general point of view is given in Figure 1. The subproblem of pattern representation involves primarily the selection of representation. The subproblem of decision-making involves primarily the selection of decision criterion or similarity measure. Other approaches include template-matching [13], problem-solving models [14], category theory [15] and relation theory [16].

In the template-matching approach, a set of templates or prototypes, one for each pattern class, is stored in the machine. The input pattern with unknown classification is matched or compared with the template of each class, and the classification is based on a preselected matching criterion or similarity measure (e.g., correlation). In other words, if the input pattern matches the template of i th pattern class better than it matches any other templates, then the input pattern is classified as from the i th pattern class. Usually, for the simplicity of the machine, input patterns and the templates are represented in their raw-data form, and the decision-making process is nothing but matching the unknown input to each template.

The template-matching approach has been used in some existing printed-character recognizers and bank-check readers [13,19]. The disadvantage of this approach is that it is sometimes difficult to select a good template for each pattern class, and to define an appropriate matching criterion. This difficulty is especially remarkable when large variations and distortions are expected in the patterns under study. Recently, the use of flexible template-matching or "rubber mask" techniques has been proposed [17].

1.2 Decision-Theoretic Approach

In the decision-theoretic approach, a pattern is represented by a set of N features or an N -dimensional feature vector, and the decision-making process is based on a similarity measure which, in turn, is expressed in terms of a distance measure or a discriminant function. In order to take noise and distortions into consideration, statistical and fuzzy-set methods have been proposed [50]. The characterization of each pattern class could be in terms of an N -dimensional class-conditional probability density function or a fuzzy set, and the classification (decision-making) of patterns is then based on a (parametric or nonparametric) statistical decision rule or (fuzzy) membership function. A block diagram of a decision-theoretic pattern recognition system is given in Figure 2.

It should be noted that the template-matching approach could be regarded as a special case of the decision-theoretic approach. In such a case, each pattern is represented by a feature vector, and the decision-making process is based on a simple similarity (matching) criterion such as the use of correlation.

Applications of decision-theoretic pattern recognition include character recognition [13,18,19], biomedical data analysis and diagnostic

decision-making [20-22], remote sensing [18,23], target detection and identification [3,24], failure analysis and diagnosis of engineering systems [25,26], machine parts recognition and inspection in the automation of manufacturing processes [27-30], processing of seismic waves [24], modeling of socio-economic systems [31], and archaeology (classification of ancient objects) [32].

1.3 Syntactic Approach

In the syntactic approach, a pattern is represented as a string, a tree or a graph of pattern primitives and their relations. The decision-making process is in general a syntax analysis or parsing procedure. Special cases include the use of similarity (or distance) measures between two strings, two trees, or two graphs [33]. A block diagram of a syntactic pattern recognition system is given in Figure 3.

Conventional parsing requires an exact match between the unknown input sentence and a sentence generated by the pattern grammar. Such a rigid requirement often limits the applicability of the syntactic approach to noise-free or artificial patterns. Recently, the concept of similarity measure between two sentences and between one sentence and a language has been developed. Parsing can be performed using a selected similarity (a distance measure or a likelihood function), and an exact match becomes unnecessary. Such a parsing procedure is called "error-correcting" parsing [34].

It should be noted that the template-matching approach could also be regarded as a special case of the syntactic approach. In such a case, each pattern is represented by a string (or tree, or graph) of primitives and the decision-making process is based on a similarity or distance measure between

two strings (or two trees, or two graphs).

Applications of syntactic pattern recognition include character recognition [35-37], waveform analysis [36,38,39], speech recognition [36,40], automatic inspection [41,42], fingerprint classification and identification [36,43], geological data processing [44], target recognition [45], machine part recognition [36,46] and remote sensing [36].

There are at least four ways to mix the decision-theoretic approach and the syntactic approach. They are: (i) decision-theoretic followed by syntactic approach, (ii) use of stochastic languages, (iii) stochastic error-correcting syntax analysis, and (iv) matching of stochastic graphs. In the following sections, we briefly describe each of these mixed approaches.

1.4 Decision-theoretic followed by syntactic approach

In this approach, pattern primitives are recognized by a decision-theoretic method and pattern structures are analyzed by a syntactic method. For example, in speech recognition, speech wave segments can be recognized by a decision-theoretic method. Strings of these segments, characterized by a set of syntax rules, provide the final description of continuous speech waveforms [18,36,47]. Similarly, such a hybrid approach can be used for EEG analysis [39]. In LANDSAT data interpretation, each pixel in a LANDSAT image can be classified by a decision-theoretic method (e.g., the maximum-likelihood classification rule) on the basis of the four-band spectral measurement. Structural (or spatial) relations among various pixels can be described by a syntactic method. Specifically, the structure of highways (or rivers) can be represented by trees with "concrete-like" or water pixels and characterized by a tree grammar. Consequently, the recognition of highways from all concrete-like pixels can be easily accomplished by using a

tree automaton [18,48]. The recognition of rivers from all the pixels classified as water can be similarly performed.

Recently, a shape recognition procedure with two types of primitive has been proposed [49]. The two primitives, curve primitive and angle primitive, are described by attributes and recognized by a decision-theoretic method. Strings of curve and angle primitives are used to represent the outer boundaries of an object with different starting points, and are characterized by a set of attributed syntax rules. Recognition of object shapes is accomplished by parsing the strings describing object boundaries with respect to the syntax rules. The structural or syntactic information contained in the syntax rules is, in fact, used to improve the primitive recognition accuracy. In other words, primitive recognition and structural analysis (or parsing) are carried out in one stage rather than one following the other in two separate stages. With the addition of error-correcting technique to the attributed shape grammar, such a hybrid approach can be used for recognition of distorted and partial shapes [65].

1.5 Use of stochastic languages

In order to describe noisy and distorted patterns under ambiguous situations, the use of stochastic languages has been suggested [10]. With the probabilities associated with grammar rules, a stochastic grammar generates sentences with a probability distribution. The probability distribution of the sentences can be used to model the noisy situations.

A stochastic grammar is a four-tuple $G_S = (V_N, V_T, P_S, S)$ where V_N is a finite set of nonterminals, V_T is a finite set of terminals, $S \in V_N$ is the start symbol, and P_S is a finite set of stochastic productions. For a stochastic context-free grammar, a production in P_S is of the form

$$A_i \xrightarrow{p_{ij}} \alpha_j, A_i \in V_N, \alpha_j \in (V_N \cup V_T)^*$$

where p_{ij} is called the production probability. The probability of generating a string x , called the string probability $p(x)$, is the product of all production probabilities associated with the productions used in the generation of x . The language generated by a stochastic grammar consists of the strings generated by the grammar and their associated string probabilities.

By associating probabilities with the strings, we can impose a probabilistic structure on the language to describe noisy patterns. The probability distribution characterizing the patterns in a class can be interpreted as the probability distribution associated with the strings in a language. Thus, statistical decision rules can be applied to the classification of a pattern under ambiguous situations (for example, use the maximum-likelihood or Bayes decision rule). A block diagram of such a recognition system using maximum-likelihood decision rule is shown in Figure 4. For a given stochastic finite-state grammar G_S , we can construct a stochastic finite-state automaton to recognize only the language $L(G_S)$ [10]. For stochastic context-free language, stochastic syntax analysis procedures are in general required. Because of the availability of the information about production probabilities, the speed of syntactic analysis can be improved through the use of this information. Of course, in practice, the production probabilities will have to be inferred from the observation of a relatively large number of pattern samples. When the imprecision and uncertainty involving in the pattern description can be modeled by using the fuzzy set theory, the use of fuzzy language for syntactic pattern recognition has recently been suggested [50].

1.6 Stochastic Error-Correcting Syntax Analysis

Recently, error-correcting syntax analysis has been proposed for the recognition of noisy and distorted patterns [33,51]. Referring to Fig. 3, a segmentation error can be represented by a deletion or insertion of a primitive in a sentence. A primitive recognition error can be expressed as a substitution of one primitive by another. With the introduction of probabilities of substitution, deletion and insertion errors, a stochastic model of syntax errors can be formulated. Using this model, the probability of deforming a sentence x to a sentence y , $q(y|x)$ can be computed. The maximum-likelihood error-correcting parsing algorithm^{††} is to search for a sentence x , $x \in L(G_s)$ such that

$$q(y|x) P(x) = \max_z \{q(y|z) p(z) \mid z \in L(G_s)\}$$

where $p(z)$ is the probability of generating z by the stochastic (pattern) grammar G_s . The term of $q(y|x) p(x)$ is called the probability that a sentence y is an error-deformed sentence of $L(G_s)$ and is denoted as $q(y|G_s)$.

By adopting the method of constructing covering grammars used by Aho and Peterson [34], we can construct a stochastic error-induced grammar from the original stochastic context-free (pattern) grammar to accommodate the stochastic deformation model. A modified Earley parser for the stochastic error-induced grammar is proposed to implement the search of the most likely error correction [51]. A more general deformation model (including the use of attributed grammars) and its corresponding Bayes error-correcting recognition system has recently been reported [52,66,67].

1.7 Matching of Stochastic Graphs

Relational graphs are used in syntactic pattern recognition to represent the structural information of patterns [10]. The nodes in a relational graph denote subpatterns and pattern primitives, and the branch between two nodes represents the relation between subpatterns and/or primitives. Recently, Tsai and Fu [53] have proposed to extend the stochastic deformation model described in Section 1.6 to error-correcting graph matching. Attributed relational graphs for syntactic pattern recognition are first defined. A stochastic deformation model for attributed relational graphs is then formulated. Only the case where the deformation does not affect the structure of the underlying unlabeled graph but only corrupts the information contained in the primitive and relations is considered. Such a deformation is called graph-preserved deformation. Pattern deformation probabilities can be calculated from primitive deformation and relation deformation probabilities. An ordered-search algorithm is proposed for determining the maximum-likelihood error-correcting isomorphisms of attributed relational graphs.

1.8 Remarks

The decision-theoretic followed by syntactic approach has been the most popular hybrid approach. The approach is simple to apply. However, noise and distortions are only considered at the local or primitive level. Segmentation error and structure distortion are not taken into consideration. The approach of using stochastic languages can certainly take care of noise and distortion at both primitive and structure levels, particularly, when the primitives are recognized by decision-theoretic methods. Practical applications include ECG interpretation and fingerprint classification

[54,55]. Unfortunately, a large number of training samples is often required to accurately infer the production probabilities. Segmentation and primitive recognition errors are explicitly considered in error-correcting syntax analysis. Probabilities for different errors can be estimated (or subjectively assigned) from the performance evaluation of segmentation and primitive recognition devices. One application of this approach is the recognition of spoken words and phrases [56]. In practice, parsing time may need to be sped up by using sequential or parallel parsing techniques [57,58]. Attributed grammars can be used to provide both syntactic and semantic information for pattern description [10,49]. A syntactic-statistical approach to pattern recognition based on attributed grammars has recently been proposed [66]. Attributed relational graphs are regarded as a more general model in describing two and three dimensional patterns. It is anticipated that the speed of error-correcting graph isomorphisms is rather slow. The use of parallel processing could be one way to speed up the procedure. The practical utility of this approach still needs to be tested.

The idea of using hybrid approaches in solving practical pattern recognition problems is not new [10,59-64]. In practice, only the decision-theoretic followed by syntactic approach can be easily applied. There is certainly a need of further studies on other possibilities of mixing the decision-theoretic and the syntactic approaches.

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Fig. 1. A general pattern recognition system.

Fig. 2. Block diagram of a decision-theoretical pattern recognition system.

Fig. 3.

Fig. 4. Maximum-Likelihood Syntactic Recognition System.

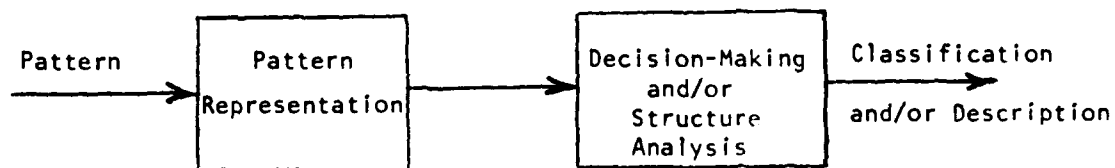


Fig. 1. A general pattern recognition system.

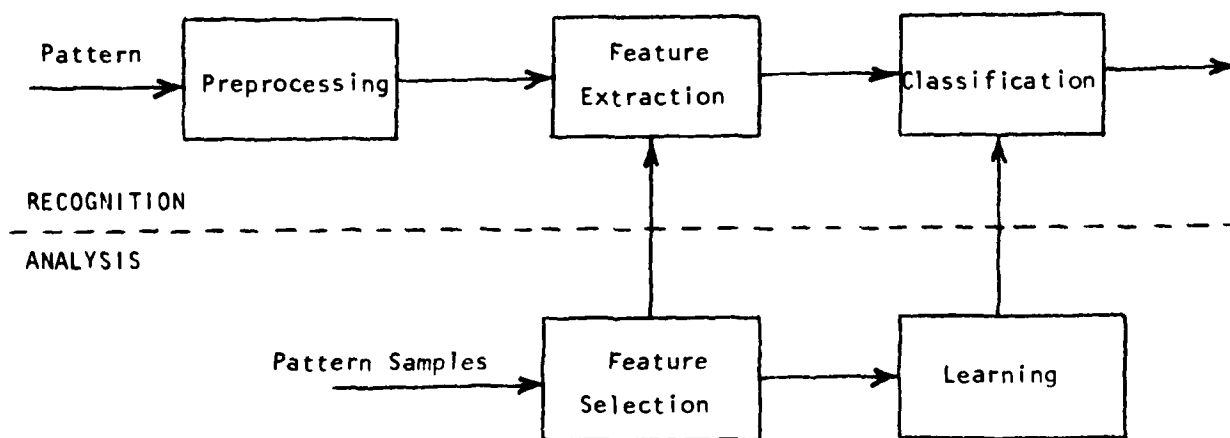


Fig. 2. Block diagram of a decision-theoretical pattern recognition system.

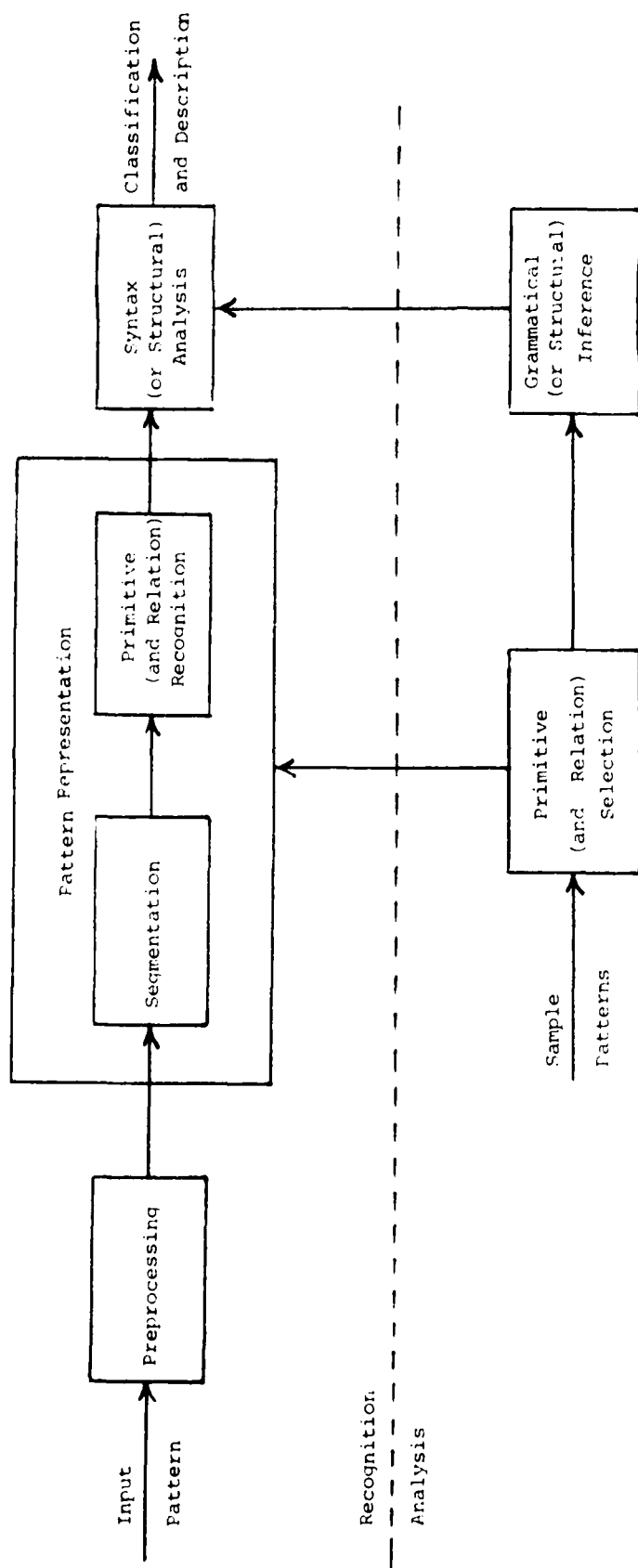


Fig. 2. Block diagram of a syntactic pattern recognition system.

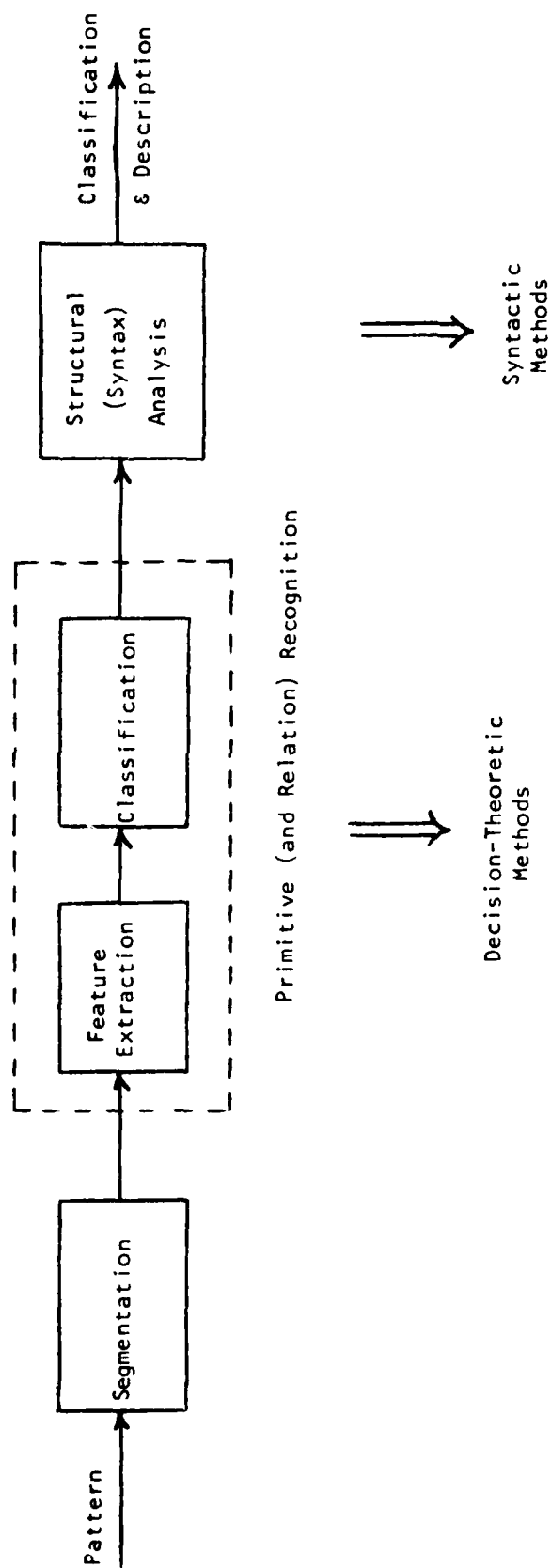


Fig. 4. Decision-theoretic followed by syntactic approach.

CHAPTER II
AUTOMATIC INSPECTION BY LOTS IN THE
PRESENCE OF CLASSIFICATION ERRORS

2.1 Introduction

Automatic inspection of manufactured products is a very important application area of pattern recognition. The multifold goals of automating this particular aspect of industrial production include raising the standard of quality control by improving the reliability of the existing inspection channels, speeding up the inspection process to keep up with the increasing production outputs due to mechanization and automation of industrial processes, relieving the human element in the inspection process from carrying out repetitive and boring tasks and, last but not least, minimizing the cost of quality control. To date a number of promising applications of pattern recognition techniques to various automatic inspection problems have been reported. In particular, methods for automatically inspecting reed switches have been described by Jarvis [1] and Van Daele et al [2]. Automatic systems for inspection of printed circuit boards [3] and LSI circuit masks [4] have recently been developed. Pattern recognition techniques have been applied to the problem of inspecting pharmaceutical products, [5] moving metal surfaces, [6] gas meters [7] etc.

Ideally, in quality control one would like to aim at inspecting every single item manufactured. In practice, however, 100% inspection is not always economically feasible even assuming an advanced stage of automation and it then becomes necessary to control the quality of a small sample of these items in each lot. On the basis of the number of defective items in the sample set a decision regarding acceptance or rejection of the whole lot is then made.

The philosophy of the quality control by lots is to reduce inspection costs by finding the minimum sample size required to ensure that each lot of products meets the quality standards specified by the consumer while keeping at a low level the manufacturer's risk of having to inspect all the items in any lot of acceptable quality or even of having to discard these items. The design of a two-sided acceptance sampling plan for this purpose is based on a family of operating characteristics which define the probability of accepting a lot of a given size as a function of the rate of defective items in the lot. The actual rate of defective items in the sample taken from the lot serves as the parameter of the family of these functions.

Quality control by lots is a long established approach to industrial inspection with a well developed methodology [8,12]. Unfortunately, the existing acceptance sampling plan design techniques are applicable only under the assumption that the classification of individual items in the sample set into the categories of defective and non-defective products is error free. When pattern recognition systems are employed to inspect individual items this assumption is not necessarily satisfied. The presence of classification errors affects the probability distribution of defective items in the sample set which has to be taken into account when designing an acceptance sampling scheme.

The effect of classification errors on acceptance sampling plans has recently been studied by Kittler and Pau [9], who considered the case where the a priori probabilities of the categories of defective and non-defective items in the lot differ from those of the mother population. They derived a system of operating characteristics which are an essential prerequisite for the design of a suitable acceptance sampling plan. In contrast to the conventional approach, the operating characteristics in their method are

parameterized in terms of the rate of items classified as defective rather than the actual rate of defective products, which is unknown. In this chapter it is assumed that the mixture probability distribution of items in a given lot has an arbitrary form which cannot be functionally related to the mother mixture population. Such a situation can arise in an environment with rapidly changing conditions in the manufacturing process or of the raw material used. It will be shown in Section 2.3 that in this case the probability distribution of classification errors cannot be predetermined. Consequently, it is not possible to obtain the operating characteristic which is required for the design of a conventional two-sided acceptance sampling plan (plan satisfying both the consumer's and manufacturer's specifications). Instead, in Section 2.4, a new quality control procedure is proposed which guarantees the product quality levels specified by the consumer. First, however, in Section 2.2, the model considered and the essential mathematical preliminaries will be introduced.

2.2 Preliminaries

Let $x = [x_1 \dots x_p]^T$ be a p -dimensional pattern feature vector representing an item to be inspected, with T denoting the transpose. We denote the classes of non-defective and defective products by ω_1 and ω_2 respectively. Further let the pattern representation space be partitioned into non-overlapping regions S_1 and S_2 associated with classes ω_1 and ω_2 . Then any pattern vector x in the region S_i will be considered as belonging to class ω_i , i.e. the decision rule determining whether x represents a good or defective item can be stated

$$\text{assign } x \text{ to } \omega_i \text{ if } x \in S_i. \quad (1)$$

The regions S_i could be determined using standard pattern classification learning algorithms [10] on the basis of the information conveyed by a training data set with labelled samples. Alternatively (and this is often the case in quality control) regions S_i are defined by prespecifying tolerances on the product characteristics as embodied by feature measurements x_j , $j = 1, 2, \dots, p$.

In the following we assume that the a posteriori class probability functions $P(\omega_i|x)$ are known at every x . This assumption implies that either the physical processes involved in generating patterns from classes ω_1 and ω_2 can be modelled with a sufficient accuracy or the training data set is large enough to allow functions $P(\omega_i|x)$ to be estimated with negligible bias and variance.

As pointed out in the introductory section, in the model considered in this paper it is assumed that the items in a lot are drawn from a mixture probability distribution characterized by a density function $\beta(x)$. Then the classification error of type I giving the rate of samples from class ω_1 being assigned to class ω_2 by decision rule (1) is defined as

$$e_1 = \int_{S_2} P(\omega_1|x) \beta(x) dx. \quad (2)$$

Similarly, the classification error of type II is given by

$$e_2 = \int_{S_1} P(\omega_2|x) \beta(x) dx. \quad (3)$$

The rate of samples classified to ω_i , \hat{c}_i , is given

$$\hat{c}_1 = \int_{S_1} \hat{p}(x) dx, \quad (4)$$

while the true rate of defective and non-defective items \bar{P} and P_n can be written respectively as

$$\bar{P} = \int P(\omega_2|x) \hat{p}(x) dx \quad (5)$$

and

$$P_n = \int P(\omega_1|x) \hat{p}(x) dx. \quad (6)$$

Note that the rate of items classified as defective by our decision making system can be expressed as

$$\hat{c}_2 = \int_{S_2} P(\omega_1|x) \hat{p}(x) dx + \int_{S_2} P(\omega_2|x) \hat{p}(x) dx. \quad (7)$$

Utilizing equations (2), (3) and (5) we get

$$\hat{c}_2 = e_1 + \bar{P} - e_2. \quad (8)$$

As in the case of the model discussed in [9], the only observable quantity in equation (8) is \hat{c}_2 . However, in contrast to that model, here the probability distributions of errors e_1 and e_2 cannot be approximated by appropriate binomial distributions. The reason for this is that the expected value of e_1 which could be used as the parameter of an approximating binomial distribution is not known. Consequently the approach to designing an acceptance sampling plan for quality control proposed in [9] cannot be adopted. In the following section the probability distribution of the rate of defective items in the sample set taken from a lot will be derived. This distribution will then be used as a basis of a new quality control scheme

proposed in Section 2.4.

2.3 The Probability Distribution of the Rate of Defective Items

It was shown in [9] that in order to check the quality of a given lot of products in the case where the probability structure underlying the distribution of items in a lot differs from that of the training data only in the a priori probabilities of classes ω_1 and ω_2 , it was sufficient to observe the realization of variable \hat{e}_2 and compare it with the predetermined threshold. Moreover, detailed knowledge of the a posteriori probabilities of classes ω_1 and ω_2 for each element x of the test set was not required. In the case of the model considered here the situation is somewhat different. We can also observe \hat{e}_2 by simply examining the position of pattern x in the test set with respect to region S_i . However, since $\beta(x)$ is assumed to be of a non-parametric form and in general, changing from one lot to another, the probability distribution of classification errors must be determined for each test set separately. This implies that the operating characteristics cannot be precomputed and an alternative strategy must be adopted. Moreover, in order to determine the probability distribution of errors, it must be possible to observe the class a posteriori probabilities for every x . It is apparent that from the computational point of view any quality control procedure for the present model will be considerably more involved.

Let us denote the actual rate of misclassified patterns from class ω_{3-i} , $i = 1, 2$, by τ_{3-i} (realization of e_{3-i}). It has been shown elsewhere [11] that the distribution function of τ_{3-i} is given as

$$g(\tau_{3-i} = k | \hat{c}_i) = (1/k) \sum_{j=1}^k \{(-1)^{j-1} g(\tau_{3-i} = k - j | \hat{c}_i)\}$$

$$\times \sum_{t=1}^{n(1-\hat{c}_i)} \left[\frac{1 - P(\omega_i | x_t)}{P(\omega_i | x_t)} \right]^j, \quad k = 1, 2, \dots, n - \hat{c}_i n, \quad (9)$$

with

$$g(\tau_{3-i} = 0 | \hat{c}_i) = \prod_{t=1}^{n-n\hat{c}_i} [1 - P(\omega_i | x_t)].$$

It cannot be over emphasized that the probability distribution in (9) is correct only under the assumption that $P(\omega_i | x)$ is known exactly. In practice this assumption will not be satisfied. However, here we assume that the cardinality of the training data set is large enough so that $P(\omega_i | x)$ can be estimated with a sufficient precision. The alternative would be to take the probability distribution of estimates of $P(\omega_i | x)$ into account in the analysis. However, from the point of view of computational complexity, this solution would make the procedure proposed impracticable.

The probability distribution of \bar{P} in equation (8) is, of course, given by the hypergeometric distribution. Since we know the distributions of all the quantities appearing in equation (8) we can now evaluate the probability that given the number of defective in the lot, d , \hat{c}_2 will take on a particular value \hat{c}_2^* . This probability is given as the sum of the probabilities of occurrence of each triplet \bar{P}^* , τ_1^* and τ_2^* yielding \hat{c}_2^* , i.e.

$$\begin{aligned} \Pr(c_2 = \hat{c}_2^* | d) = & \sum_{\bar{p}, \tau_1^*, \tau_2^*} \Pr(\bar{p} = \bar{p}^* | d) \Pr(\tau_1 = \tau_1^* | \hat{c}_2^*) \Pr(\tau_2 = \tau_2^* | 1 - \hat{c}_2^*) \\ & \times \delta(\hat{c}_2^* - \bar{p}^* - \tau_1^* + \tau_2^*) \end{aligned} \quad (10)$$

$$\bar{p}^* = (k/n) \quad k = 1, 2, \dots, \min[n, d]$$

$$\tau_1^* = (k/\hat{c}_2^* n) \quad k = 0, 1, 2, \dots, \hat{c}_2^* n$$

$$\tau_2^* = (k/\hat{c}_1^* n) \quad k = 0, 1, 2, \dots, \hat{c}_1^* n,$$

where $\delta(\cdot)$ is the Kronecker delta function.

We have thus obtained an expression for calculating the probability that given d the classification system will assign exactly $\hat{c}_2 n = \hat{c}_2^* n$ items into the class of defective. Note however, that function $\Pr(\hat{c}_2 = \hat{c}_2^* | d)$ of argument d is not a probability distribution function. Further, it would be more convenient to be able to say what is the probability that the lot contains exactly d^* bad products given $\hat{c}_2 = \hat{c}_2^*$ rather than work with the probability of observing \hat{c}_2^* under the various hypotheses. Invoking the Bayes formula for calculating conditional probabilities we get

$$\Pr(d = d^* | \hat{c}_2^*) = \frac{\Pr(\hat{c}_2 = \hat{c}_2^* | d) \Pr(d = d^*)}{\Pr(\hat{c}_2 = \hat{c}_2^*)} \quad (11)$$

We shall assume that a priori probability of occurrence of any value of d is equally likely. Since there are $N + 1$ possible values d can assume ($d =$

0,1,2,...,N) then

$$\Pr(d = d^*) = \frac{1}{N+1}. \quad (12)$$

The unconditional probability $\Pr(\hat{c}_2 = \hat{c}_2^*)$

$$\Pr(\hat{c}_2 = \hat{c}_2^*) = \frac{1}{N+1} \sum \Pr(\hat{c}_2 = \hat{c}_2^* | d). \quad (13)$$

Thus we can write

$$\Pr(d = d^* | \hat{c}_2^*) = \frac{[\Pr(\hat{c}_2 = \hat{c}_2^* | d)]}{\left[\sum_{d=0}^N \Pr(\hat{c}_2 = \hat{c}_2^* | d) \right]}. \quad (14)$$

Using expression (14) we can determine the conditional probability distribution of random variable d given \hat{c}_2^* and, naturally, the cumulative distribution

$$\mu(d) = \sum_{d^*=0}^d \Pr(d = d^* | \hat{c}_2 = \hat{c}_2^*). \quad (15)$$

2.4 Acceptance Sampling Strategy

The curve $\mu(d)$ defines at every point d the probability that, assuming \hat{c}_2^* has been observed, there are at most d defective items in the lot. On the other hand the curve $1 - \mu(d)$ defines for every d the probability that the lot contains more than d defective products. Based on these observations we can now propose a quality control test as follows.

Hypothesis

Null hypothesis H_0 (accept the lot)

number of defective $d \leq d_T$

Alternative hypothesis H_1 (reject the lot)

number of defective $d > d_T$,

where d_T is a given threshold.

Accept H_0 if $1 - \mu(d_T) < \beta$, otherwise reject H_0 . (16)

We shall now summarize the proposed quality control scheme.

1. Classify elements of the test set of size n taken from a lot to obtain \hat{c}_i^* , $i = 1, 2$.
2. Determine the probability distributions of errors e_1 and e_2 using equation (9).
3. Evaluate $\Pr(\hat{c}_2 = \hat{c}_2^* | d)$ for all d according to equation (10).
4. Determine the cumulative distribution $1 - \mu(d)$ in equation (15).
5. Apply hypothesis test (16).

A few comments are in order here. First of all, there is a difference between the quality control concepts employed here and in [9]. In the case of the model discussed in [9] the manufacturer guarantees that the probability of a lot with d_T or more defective items passing through the quality control does not exceed β , i.e. $\Pr(H_0 \text{ accepted} | d > d_T) \leq \beta$. On the other hand, in the present model the manufacturer ensures that the probability of accepted lots containing d_T or more defective items is less than β , i.e. $\Pr(d \geq d_T | H_0 \text{ accepted}) \leq \beta$. Note, however, that for any model we have

$$\Pr(H_0 \text{ accepted} | d) \geq d_T = \Pr(d \geq d_T | H_0 \text{ accepted}) \frac{\Pr(H_0 \text{ accepted})}{\Pr(d \geq d_T)} . \quad (18)$$

Let us consider the relationship (18) in more detail. According to equation (12) we have

$$\Pr(d \geq d_T) = \frac{N - d_T + 1}{N + 1}$$

Further, under the assumption that the a priori probability of accepting H_0 equals the a priori probability of the lot containing $d < d_T$ defective items, i.e.

$$\Pr(H_0 \text{ accepted}) = \frac{d_T}{N + 1} . \quad (19)$$

Equation (18) implies

$$\Pr(H_0 \text{ accepted} | d \geq d_T) \leq \beta \frac{d_T}{N - d_T + 1} < \beta ,$$

$$\text{provided } d_T \leq N - d_T + 1 . \quad (20)$$

Thus the quality control scheme developed automatically satisfies the consumer's risk specifications. It cannot be overemphasized however that the parallel between these two models can be drawn only under the assumption of the validity of equation (19) and of the particular model for the distribution $\Pr(d = d^*)$.

The main shortcoming of the proposed sampling scheme is the lack of any guidelines for choosing the size of the test set, n . In principle, the acceptance sampling plan should be applied for several (monotonically increasing) values of n with the quality check terminating when $1 - \mu(d_T)$ remains

constant.

The computational burden of such a control scheme could be eased by approximating distribution (9) to the binomial distribution with parameter \bar{e}_i

$$\bar{e}_i = \frac{1}{n - n\hat{e}_i^*} \sum_{t=1}^{n - n\hat{e}_i^*} [1 - P(\omega_i | x_t)] \quad (21)$$

$$x_t \rightarrow \omega_i.$$

If the accuracy afforded by this approximation were deemed to be satisfactory it would be possible to precompute a set of parametric acceptance sampling plans as in (9) in the form of a look up table, giving appropriate values of \hat{e}_{2T} for the whole spectrum of combinations of \bar{e}_i , $i = 1, 2$. Since \bar{e}_i depends on n , the acceptance threshold would have to be determined for the minimum value of n as a function of \bar{e}_1 and \bar{e}_2 satisfying the given quality control specification.

2.5 Conclusions

A pattern recognition system for the inspection of products by lots has been studied. It has been shown that in the presence of classification errors the existing acceptance sampling plans cannot be used. An alternative quality control procedure has been developed for the model assuming an arbitrary distribution of patterns in the lot. Computationally the scheme is very demanding. Some simplification can be achieved by approximating the actual probability distributions of classification errors with the binomial distributions of having identical expected values.

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CHAPTER III
VISUAL SCREENING OF INTEGRATED CIRCUITS FOR METALLIZATION FAULTS
BY PATTERN ANALYSIS METHODS

3.1 Introduction

As the complexity of integrated circuits (IC's) increases, the testing problem becomes more and more acute in terms of final production yield and IC costs [1].

- metallization defects (open or short circuits, scratches, migration, corrosion)
- wire and die bonds (open, shorted, fatigued)
- process faults, esp. oxide pinholes and diffusions
- surface defects and loose particles
- die cracks, dirty photomasks
- external leads
- dielectric failures
- packaging defects and seals
- thermal mismatch
- violation of design rules

The usual testing procedure includes a suitable combination of the following basic testing processes [1].

- pre-cap and external visual inspection, and X-ray inspection
- electrical testing (pre-cap and after packaging)
- environmental testing, especially temperature cycling or shocks.

One of the more fundamental constraints about current IC testing procedures is the fact that those listed above are implemented in sequence, at many different stages of the manufacturing process [4]. This limitation becomes even more severe if lot inspection procedures are totally discarded at

some of these stages, in order to achieve 100% testing throughout the manufacturing process.

(A) Pre-cap visual inspection: The thorough visual inspection before the chip is encapsulated is designed to eliminate unreliable circuits from further processing. The visual inspection is generally carried out by human operators with the aid of high power and low power microscopes; it includes essentially the leads, die and wire bonding, and the topology of the chip. Rigorous adherence to 100% pre-cap visual inspection prior to encapsulation is also essential to weed out potentially unreliable circuits that would otherwise pass all other screening tests. While the effectiveness of pre-cap visual testing is high, the cost and time related to human operators is too high.

(B) Electrical testing: Test patterns are usually reserved by the IC layout to provide for convenient areas for contact probing (in addition to permanent leads). These areas are included within the actual circuit die area to assure the necessary matching of characteristics. In addition to providing locations for probing with minimum damage to the actual circuits, and minimum electrical interference, the test pattern may be designed to provide for components which amplify the signals to be measured [5,6] (Fig. 1). A prerequisite to the use of the test patterns through probing is the proper alignment of the IC. Electrical testing does not in general lead to fault location on the IC, and undetectable failure modes may exist especially when only limited testing is applied.

(C) Thermal cycling and shocks: These tests will weed out many future faults or defects not apparent to visual inspection, in addition to helping localize them. They are e.g. recommended for crystal imperfections, cracked dies, oxide pinholes, oxide shorts, passivation defects, opening of thermal seals, poor wire bonds. Thermal testing is usually unattended, but lengthy and costly.

(D) Integrated pre-cap testing: In order to speed up and automate the pre-cap testing stage, this chapter proposes the concept of integrated pre-cap testing and pattern analysis methods to implement it. These methods are developed to allow for a direct interaction with IC design tools. The goal is defect detection and possible localization by automated IC image analysis at different wavelengths, while electrical testing and some thermal tests are carried out, without any test bench transfer. The methods proposed are not restricted to periodic structures (although they would be simplified by such assumptions), nor to only vertical/horizontal etchings. They allow for possibly large geometric or topological deformations, and defect scattering, and are not based on deformations of reference layouts as usually the case in the literature. Also, the topological layout is explicitly allowed to be context sensitive as in reality, as opposed to context free assumptions.

3.2 Automatic Visual Inspection of Masks and IC's

This section briefly surveys current and past research in this field.

(A) Visual and electrical testing both require test patterns on the IC; original probe-pad test patterns have been designed which contain visual alignment indicators and probe resistors [5,6]. Coarse prior mask

and IC alignment may be necessary before fine alignment via the test patterns; line-by-line scanning, or vertical and horizontal boundary detection then takes place for the estimation of the bias and the tilt angle of the mask/IC [8,9,10,23]. Coarse IC alignment is often mechanical.

(B) Once a proper fine alignment is completed, pre-cap visual inspection may get started. All existing automated methods can be divided into the following three types [11]:

- a. defect enhancement by image filtering
- b. image matching
- c. pattern matching

In this respect, it is necessary to point out the fact that most methods and systems were actually designed as extensions of printed circuit board/drawing (PCB) inspection systems, or are at least restricted to PCB inspection [12,13,14,15,16]. Consequently, the sensors used are exclusively TV or CCD line-by-line scanners [13], and many problems specific to IC's/masks have been neglected. At the same time, no advanced pattern recognition methods have been considered or developed for this IC application (inspection).

(C) Defect enhancement by image filtering is done by operating on the one-dimensional Fourier transform or Vander Lugt filter of each scan, and by detecting peaks in the transform [17,18]. This approach is restricted to strictly periodic structures, and remains sensitive to positioning errors and tolerances.

(D) Image matching consists of comparing adjacent similar chip patterns on the same die or mask, with additional comparison to reference patterns [11,19,20,24]. Defects are recorded as differences by image subtrac-

tion or by correlation [21,22]. Especially comparison on adjacent dies of $50 \mu \times 50 \mu$ windows has been investigated and is considered a standard procedure. The matching is however subject to small edge misalignment, edge coloring, misregistration, human errors, and image enhancement remains necessary.

- (E) Pattern matching is related to the production of small (5x5 or 10x10 pixels) reticles (straight segments, corners, spots). It has been shown, in the case of PCB, that only a moderate, e.g. 500, number of all possible mask patterns are needed to describe all areas in the true layout [12] for one layer at a time. These features, usually binary, are then correlated with a reference for failure detection.
- (F) In general, however, these three approaches are organized into a hierarchical inspection process, with various levels according to the resolution, area and field of view [27]. Microfaults revealed through visual or other means (X-rays), depending on the nature of the substrate, are important. It has been shown that such faults tend to cluster in groups with varying spacings, and that fault density clustering may be considered for yield predictions [4,29].

3.3 Integrated Pre-Cap Testing

This testing procedure has been defined in Section 3.1(D). At the design stage, it uses chip/mask layout by computer-aided design (CAD) with organization into cells [30,31,32]. The screening will thus be reduced to sequences of such cells, using the topological layout features in the CAD, rather than the geometrical features only. The test points are assumed to be generated and selected by CAD within each cell [33,34].

Whereas some experiments have been carried out on the comparison of adjacent dies at 3 different colours [20], we here consider a procedure using 2 different wavelengths in the visible domain and 1 or 2 wavelengths in the middle infrared (IR) domain (3 and 10 μ). In other words, IC pre-cap inspection with infrared thermography is considered, thus allowing for simultaneous visual, electrical and eventually thermal testing [35,36,37,38]. The near-IR inspection will lead to the detection of hot spots under various testing patterns. The middle-to-long IR inspection will localize many metallization and bonding defects, again under various electrical testing patterns. The choice of the wavelengths, resolution and window sizes will clearly depend on lithographic resolution, circuit density, and not least substrate properties. Although IR emissivity is affected by surface condition and substrate, excellent temperature resolutions can be obtained both on silicium and gallium arsenide [36], thus assisting failure localization when emission anomalies are observed while electrical testing is carried on. Another advantage of this procedure is to restrict, by emissivity considerations, the size of the IC cell portion activated through electrical stimuli selection [39,51], thus leading to less complex image patterns to be analyzed than in the visible domain. We will call sub-cell any such IC cell portion activated through electrical stimuli.

In the remainder of this chapter, we restrict ourselves to the visual inspection methods related to subcells, within the framework of the above procedure. The subcell images at the various wavelengths are assumed to be acquired through the optical field of view of a multi-lens microscope. The subcell images are also assumed to be thresholded and digitized on a few gray levels.

3.4 Algorithm #1: Matching Bridges in the Topological Subcell Layouts

(A) Principle: The idea of Algorithm #1 is not to match IC cell patterns, but to match critical topological elements called bridges [40,41]. The bridges are those sections of the IC subcell surface which are nonredundant for proper electrical circuit/subcell operations; in case of defects, short circuits and conductor-substrate interface anomalies also become parasite bridges (Fig. 2). The bridges are determined by Algorithm #1 operating on the graph representation G of the subcell as obtained from the IC image. A simple thinning procedure using local neighborhood relations is used to get the graph representation G of the subcell, as seen under current optical and electrical conditions. It should be noticed that this thinning procedure is far easier to implement than any parsing of the IC etching boundaries [42]. Smaller defects eliminated because of the thinning will be picked up by Algorithm #2.

(B) Graph representation of the IC subcell (Fig. 2): The n -node graph $G=(X,U)$ labeled with a path algebra P can be described by its adjacency matrix, which is the $n \times n$ matrix $A \triangleq (a_{ij})$ with entries:

$$a_{ij} \triangleq \begin{cases} l(x_i, x_j), \text{ label of } (x_i, x_j) & \text{if } (x_i, x_j) \in U \\ \phi & \text{if } (x_i, x_j) \notin U \end{cases}$$

where ϕ is the zero element of P , and U the order relation between nodes (Fig. 2, Table 1). The k -th power A^k of A can be defined in terms of the labels of paths on the graph corresponding to A , in the following way. Let S_{ij}^k be the set of all paths of order k from node x_i to node x_j on the labeled graph G of A ; then:

$$a_{ij}^k = V \{l(s); s \in S_{ij}^k\} \quad \text{where} \quad \left| \begin{array}{l} V: \text{ join operation in } P \text{ (Table 1)} \\ \cdot: \text{ product in } P \text{ (Table 1)} \end{array} \right.$$

Each element a_{ij}^k of A^k is the set of names of all simple paths of order k from node x_i to node x_j .

We shall denote the strong and weak closure of a stable matrix A by $A^* = (a_{ij}^*)$ and $\hat{A} = (\hat{a}_{ij})$, respectively. A^* is such that:

$$\begin{aligned} \forall k=0,1,\dots \quad A^{*k} &= A^{*(k+1)} \\ A^* &= E \vee \hat{A} \quad A^* = A^{(n-1)} \end{aligned}$$

$E \triangleq$ Identity matrix for \cdot in P (Table 1)

$$\hat{A} = AA^* = \bigvee_{k=1}^n A^k$$

Each element a_{ij}^* of A^* is the set of names of all the simple paths from x_i to x_j . Each element \hat{a}_{ij} of \hat{A} is the set of names of all non-null simple paths from x_i to x_j . If only binary labels are considered (binary pictures), A is the boolean adjacency matrix of the graph G ; A^* has then entries $a_{ij}^* = 1$ if there exist any paths from x_i to x_j , and $a_{ij}^* = 0$ otherwise; whereas \hat{A} has $\hat{a}_{ij} = 1$ if there exist any non-null paths from x_i to x_j , and $\hat{a}_{ij} = 0$ otherwise (see Table 1).

(C) Bridges of the subcell graph G : Let $G=(X,E)$ be the simple graph representing the subcell whose edges have distinct labels; let $H=(X,U)$ be the graph with the same nodes as G , and which has two arcs (x_i, x_j) and (x_j, x_i) between each pair of nodes x_i, x_j which are joined together by an edge on G ; on H , the arcs (x_i, x_j) and (x_j, x_i) both bear the name of the corresponding edge (x_i, x_j) on G .

An edge (x_i, x_j) of the simple graph G is called a bridge of G , if in the graph obtained from G by removing this edge, the nodes x_i and x_j are not connected; in Fig. 3, f is a bridge.

(D) Determination of the bridges of G : Considering H , each entry of the closure a_{ij}^* of its adjacency matrix A , is the set of names of all the bridges between x_i and x_j . Thus, to find these bridges, we need to be able to compute \hat{A} or A^* directly. One such method is the Jordan elimination method which can be applied to compute the weak closure \hat{A} . \hat{A} is the least solution of the equation $Y = AY \vee B$, if we set $A^{(0)} = B^{(0)} = A$, because then $\hat{A} = B^{(n)}$. The steps of the algorithm are the following:

$$\begin{cases} B^{(k)} = Q^{(k)} * B^{(k-1)} \\ \hat{A} = B^{(n)} \quad k=1, 2, \dots, n \quad A^{(0)} = B^{(0)} = A \end{cases}$$

$$Q^{(k)*} \triangleq \begin{bmatrix} E & B_{12}^{(k-1)} & B_{22}^{(k-1)*} & \phi \\ \phi & & B_{22}^{(k-1)*} & \phi \\ \phi & B_{32}^{(k-1)} & B_{22}^{(k-1)*} & E \end{bmatrix}$$

where:

- the B_{lk} blocks are made of elements b_{ij} , as specified below;
- the closure of an element is defined in a similar way to the closure of a matrix;

$$b_{ij}^{(k)} = \begin{cases} b_{ik}^{(k-1)} \cdot (b_{kk}^{(k-1)})^* & \text{if } j=k \\ (b_{kk}^{(k-1)})^* \cdot b_{kj}^{(k-1)} & \text{if } i=k \\ b_{ij}^{(k-1)} \vee b_{ik}^{(k-1)} \cdot (b_{kk}^{(k-1)})^* \cdot b_{kj}^{(k-1)} & \text{if } i, j \neq k \end{cases}$$

Example: (Fig. 3) Here \$ is the empty label, and ϕ the zero element in G

$$A = \begin{bmatrix} \$ & a & b & c & \$ & \$ & \$ \\ a & \$ & d & \$ & \$ & \$ & \$ \\ b & d & \$ & e & \$ & \$ & \$ \\ c & \$ & e & \$ & f & \$ & \$ \\ \$ & \$ & \$ & f & \$ & g & h \\ \$ & \$ & \$ & \$ & g & \$ & i \\ \$ & \$ & \$ & \$ & h & i & \$ \end{bmatrix} \quad A^* = \begin{bmatrix} \$ & \$ & \$ & \$ & | & f & f & f \\ \$ & \$ & \$ & \$ & | & f & f & f \\ \$ & \$ & \$ & \$ & | & f & f & f \\ \$ & \$ & \$ & \$ & | & f & f & f \\ - & - & - & - & - & - & - & - \\ f & f & f & f & | & \$ & \$ & \$ \\ f & f & f & f & | & \$ & \$ & \$ \\ f & f & f & f & | & \$ & \$ & \$ \end{bmatrix}$$

Other algorithms are given in [41,43,44,45].

(E) IC testing: For each wavelength, each set of electrical stimuli, and each subcell with resulting graph G, the bridges of G are matched against those of the reference topological layout (CAD).

3.5 Algorithm #2: Computation of a figure of Merit for the IC/Mask from a Fuzzy Language Description

(A) Principle:

1) Algorithm #2 is designed to compute for each subcell a figure of merit μ which accounts both for topological and geometrical faults in the IC/mask, while still accounting for lithographic resolution and image acquisition errors. It is better suited than Algorithm #1 for the detection of irregular growth/etching boundaries, scratches, blobs, open circuits. Acceptance or reject of the IC/mask is on the basis of the figures of mer-

it of all the subcells.

2) Algorithm #2 relies on the following elements:

(a) The topological model of the fault-free subcell as a monoid V_O^* of a context-sensitive language L_O ; V_O^* is generated by the CAD design software, restricted to the subcell for which electrical testing is proceeding (all IC layers are considered).

(b) A fuzzy class membership relation ρ for each string x of symbols generated in L_O , $0 \leq \rho(x) \leq 1$. The actual value of $\rho(x)$ will be derived from the subcell image, as to enhance defects, while only taking into account those defects the sizes of which are in excess of process tolerances and optical resolution.

Example: i) If a is a "primitive" etching shape, $\rho(a)$ could be proportional to the area of each such pad on the IC, as determined by thresholding and counting of pixels. Any irregular off-shots, blobs, partial bridges, would then affect the value of $\rho(a)$.

ii) $\rho(a)$ could be defined differently for various values a , to account for errors at nodes and on the line etching elements.

(c) The recursive computation of the degree of agreement $\mu(x)$ of the actual subcell x , with the language L_O , where x is assumed to be generated by a fuzzy language derived from L_O and from the fuzzy relationship ρ ; this degree of agreement is the figure of merit for the actual layout on the IC/mask under test.

(B) Grammar G_O [47]: Let V_T be the finite set of primitives of the IC/mask layout, including primitives associated with easily detectable geometrical defects. We denote V_T^* the set of finite strings obtained by concatenation of primitives of V_T , including the null string ϕ . The language L_O is a

subset of V_T^* , specified by the CAD design; it represents all acceptable layouts of the subcell. The elements of L_0 may be generated by the grammar $G_0 = (V_N, V_T, P_0, s)$, where V_T is the set of terminals/alphabet, V_N is a set of nonterminals, $s \in V_N$ is the start symbol, and P_0 is the finite set of production rules. The elements of P_0 are rewriting rules of the form $a \rightarrow b$, where a, b are strings in $(V_T \cup V_N)^*$. These rules are those by which the IC subcell layout is obtained starting with $a=s$.

One important property of IC's/masks, often overlooked in practice, is that the corresponding grammar G_0 is context-sensitive for most circuit layouts. G_0 is said to be context-sensitive if the productions are of the form $a_1 A a_2 \rightarrow a_1 B a_2$, with a_1, a_2, B in $(V_T \cup V_N)^*$, A in V_N , $B \neq \phi$, and $s \rightarrow \phi$ allowed.

(C) Fuzzy grammar [46] $G = (V_N, V_T, \pi, s)$: G is derived from G_0 by replacing P_0 by fuzzy rewriting rules defined as $a \xrightarrow{\rho} b$, where ρ is the grade of membership of string b given a , or the figure of merit of b given a as obtained from the subcell image. G generates a fuzzy language L for which one can define the degree of properness $\mu(x)$ of any string $x \in V_T^*$, valuating to what extent it is correct w.r.t. G_0 .

(D) Figure of merit of each subcell $x \in V_T^*$:

a) Let a_1, \dots, a_m be strings in $(V_T \cup V_N)^*$, and $a_1 \xrightarrow{\rho_1} a_2, \dots, a_{m-1} \xrightarrow{\rho_{m-1}} a_m$ be productions in π of G . Then a_m is said to be derivable from a_1 in G . The string x of V_T^* representing the actual IC subcell is said to be in L iff x is derivable from s . The grade of membership $\mu(x)$ of x in L is defined as:

$$\mu(x) \triangleq \text{Sup Min } (\rho(s \rightarrow a_1), \dots, \rho(a_m \rightarrow x))$$

where the supremum is taken over all derivation chains from s to x . Consequently, the figure of merit $\mu(x)$ of subcell x is the degree of properness of the least proper link in the derivation chain generating the actual subcell x , and $\mu(x)$ is calculated on the "best" chain.

(b) G is said to be recursive iff there is an algorithm which computes $\mu(x)$ recursively. G_0 is context-sensitive, so is G . As it has been shown [48] that a fuzzy context-sensitive grammar was recursive, the figure of merit $\mu(x)$ of the subcell x on the IC can be computed recursively. For details about the design of this algorithm, see [48]. We use it here, and apply it to the sequence of measured values $\rho(a_i \rightarrow a_{i+1})$, obtained from the IC image as specified in 3.5 A, b.

3.6 Algorithm #3: Attribute Labelled Graphs

(A) Principle: In this section, we will only suggest a third class of algorithms, without any explicit derivation of them. In the case of IC/masks, probabilistic deformation mechanisms represent an insufficient formalism [49]. It is here suggested to consider instead a syntactically driven random field model, also called attribute labelled graph. Instead of looking at bridges as in Section 3.4, jumps between nodes are considered here, with each node having a label which, too, may be distorted representing a topological defect.

(B) Approach: The best current approach is the proposed error-correcting recognition system presented in Chapter 1 of this report, and in [50]. The defects are modeled as, first, a syntactic deformation of each primitive or subpattern, followed next by a local deformation. The syntactic

deformations are however assumed to be independent of the context and of the local deformation, which is sometimes inappropriate for defects such as short or open circuits. The detection is by an error-correcting isomorphism, and Bayes decision comparing the original and final graphs (Chapter 1, and [50]).

3.7 Conclusion

This chapter presents two algorithms, and one approach for automated pre-cap visual inspection in the suggested integrated testing framework. Although an experimental validation is required, this testing process and the associated algorithms are much more sophisticated than current practices and should hopefully give ideas for cost, time and yield improvements in IC manufacturing.

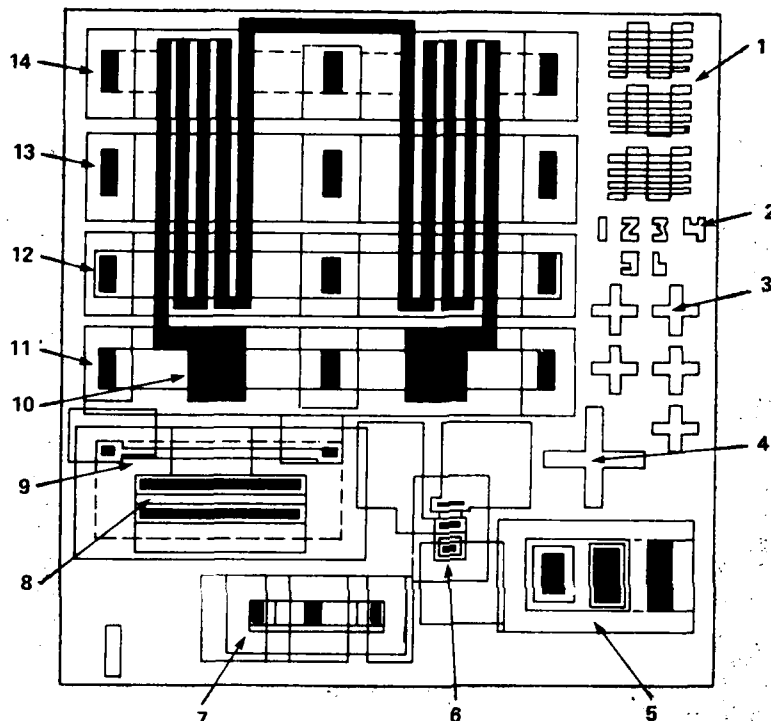
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|--|---|
| 1. RESOLUTION PATTERN
FOR 6 MASKS | 7. SQUEEZED RESISTOR
(BASE WIDTH) |
| 2. MASK IDENTIFICATION | 8. BASE-DIFFUSED RESISTOR |
| 3. PROGRESSIVE ALIGNMENT KEY
(FOR EXAMPLE, ISO-TO-BL,
BASE-TO-ISO, etc.) | 9. BASE-DIFFUSED RESISTOR |
| 4. MASTER ALIGNMENT KEY
(ALL MASKS ALIGNED TO
EACH OTHER) | 10. METALLIZATION RESISTANCE |
| 5. LARGE PROBE TRANSISTOR | 11. SHEET RESISTANCE OF EMITTER |
| 6. TYPICAL SMALL-SIGNAL
TRANSISTOR | 12. SHEET RESISTANCE OF BASE |
| | 13. SHEET RESISTANCE OF
EPITAXIAL DEPOSITION |
| | 14. SHEET RESISTANCE OF
BURIED LAYER |

FIGURE 1 : TEST AND ALIGNMENT PATTERN

FIGURE 2 : SUBCELL GRAPH LABELS

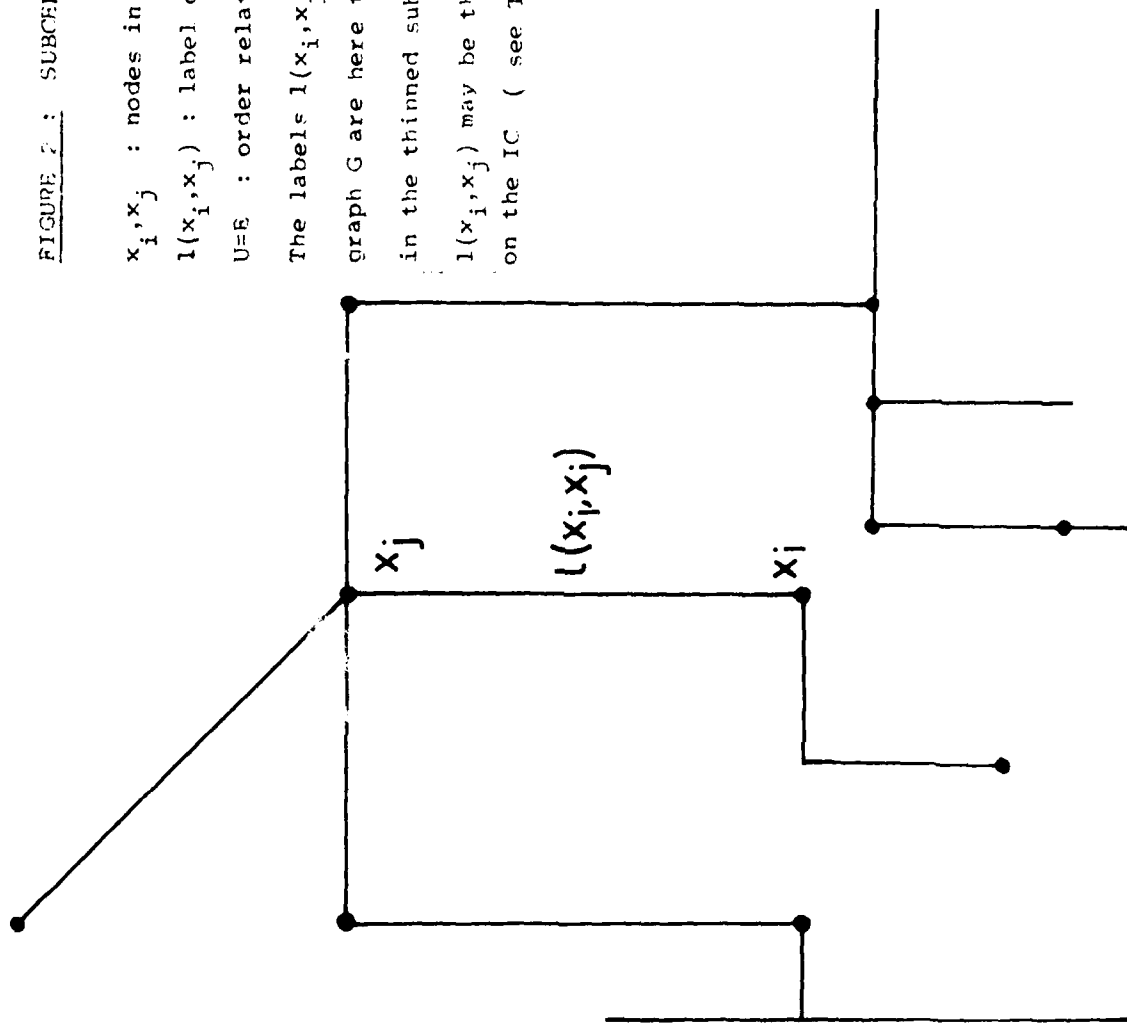
x_i, x_j : nodes in the simple(non directed)graph G

$l(x_i, x_j)$: label of the edge (x_i, x_j) in G

$U \in E$: order relation of this non-directed graph

The labels $l(x_i, x_j)$ of the edges (x_i, x_j) of the graph G are here the gray_levels of these_arcs in the thinned sub-cell image. Alternatively

$l(x_i, x_j)$ may be the physical length of this edge on the IC (see Table 1)



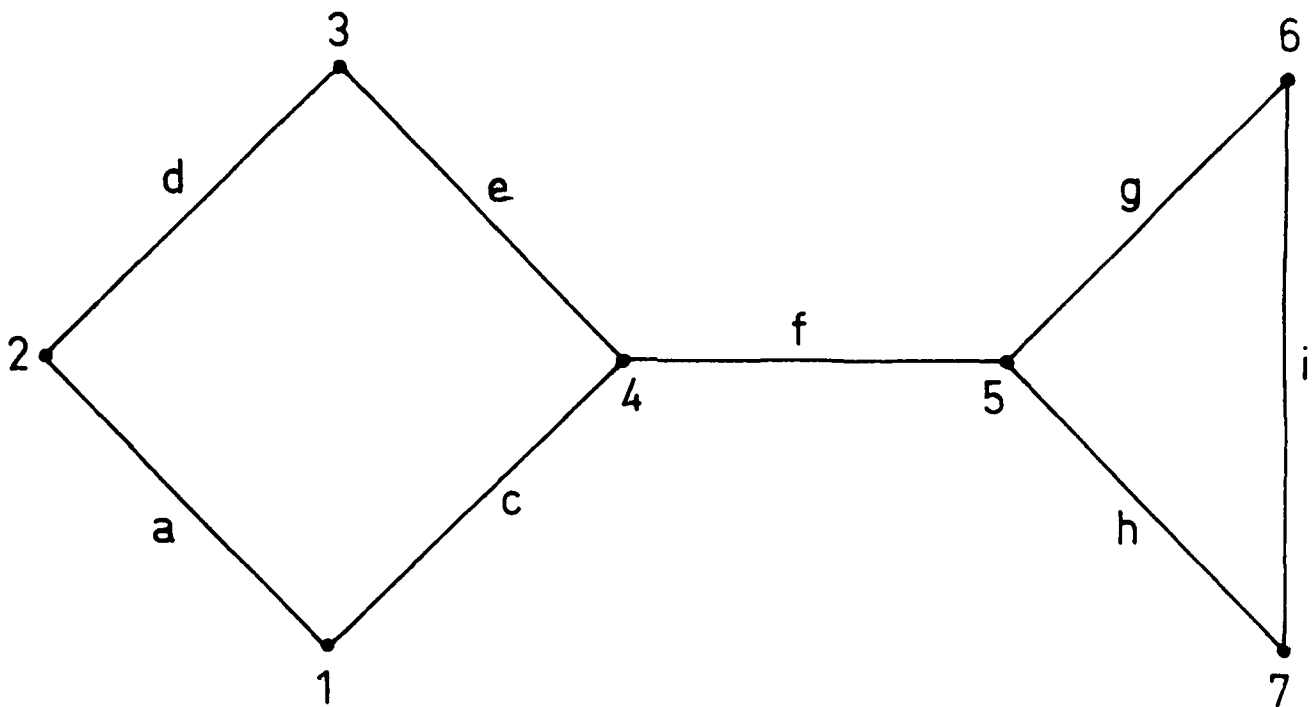


FIGURE 3 : EXAMPLE OF BRIDGE FINDING IN THE SUBCELL GRAPH G

Nodes : 1,...,7 Labels : a,b,...,i

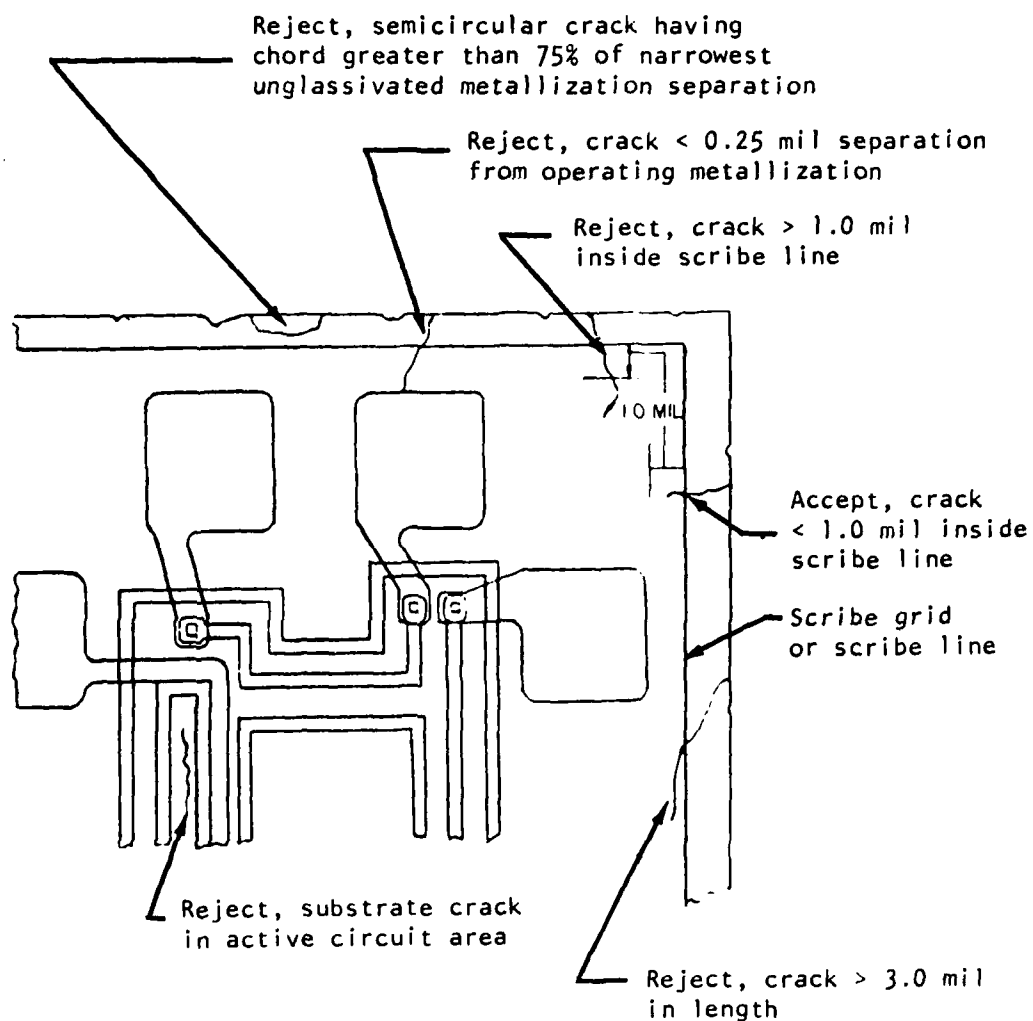


FIGURE 4: Figure 2010-25 from MIL-STD-883 which is an example of visual inspection criteria for an integrated circuit. Interpretation can be subjective.

TABLE 1 : ARC LABELS IN G AND POSSIBLE PATH ALGEBRAS P

P has the two operations : \vee (joint) and \cdot (multiplication)

ARC LABEL $l(x_i, x_j)$ AND ALGEBRA P	SIGNIFICANCE OF a_{ij} OR \hat{a}_{ij}
<p>0,1 (binary pixels)</p> <p>$x \vee y = \text{Max } (x,y)$ $x \cdot y = \text{Min } (x,y)$</p> <p>$\emptyset = 0$ $E = 1$</p>	<p>$a_{ij} = 1$ if x_j is accessible from x_i, and</p> <p>$\hat{a}_{ij} = 0$ otherwise ;</p> <p>$\hat{a}_{ij} = 1$ if there exists a non-null path from x_i to x_j, and $\hat{a}_{ij} = 0$ otherwise</p>
<p>Physical length of the edge</p> <p>$x \vee y = \text{Min } (x,y)$ $x \cdot y = x + y$</p> <p>$\emptyset = \infty$ $E = 0$</p>	<p>a_{ij} is the length of the shortest path from x_i to x_j</p>
<p>Probability of existence of the edge</p> <p>$x \vee y = \text{Max } (x,y)$ $x \cdot y = xy$</p> <p>$\emptyset = 0$ $E = 1$</p>	<p>a_{ij} is the reliability of a most reliable path from x_i to x_j</p>

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